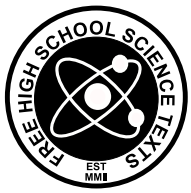

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Contents

Part V

Exercises

Chapter 46

General Exercises

1. [IEB, Nov. 2004, HG] The notation $\lfloor x \rfloor$ means the largest integer less than or equal to x. For example, $\lfloor 3{,}7 \rfloor = 3$ and $\lfloor 5 \rfloor = 5$

   A i. Show that $x = 35$ is a solution to the equation:

   $$x - \left\lfloor x^{\frac{1}{2}} \right\rfloor^2 = 10$$

   ii. Find any two other values of x that satisfy this equation.

   B Sketch the graph of $y = \lfloor x \rfloor$ for $0 \leq x \leq 4$.

2. [IEB, Nov. 2001, HG] How many digits are there *before* the decimal point in the number $(2{,}673)^{90}$? Explain your answer briefly.

Chapter 47

Exercises - Not covered in Syllabus

1. [IEB, Nov. 2001, HG]

   A Sketch two curves which will enable you to solve $|x-2| \leq |x| - 1$ graphically.

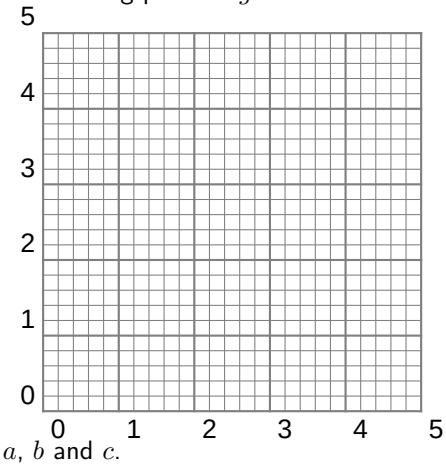
   B *Calculate* the point(s) of intersection of the two curves, and *hence*, or otherwise, write down the solution set to the inequality above.

2. [IEB, Nov. 2002, HG] Evaluate without using a calculator: $|2x-1| - |-5x|$ given that $x = -1$

3. [IEB, Nov. 2003, HG] In the figure below, two graphs are shown:

   $$f(x) = ax^2 + bx + c \qquad \text{and} \qquad g(x) = |x + c|$$

   $x = -1$ is the equation of the axis of symmetry of the parabola. f contains the points (2; 0) and (4; -8), and P is the turning point of g.

   A Find the values of a, b and c.

   B Find the length of MN if MN is perpendicular to the y-axis and MN produced passes through the y-intercept of $f(x)$ and $g(x)$.

   C Determine the equation of the graph that results when $g(x)$ is reflected about the line $x = 1$.

4. [IEB, Nov. 2003, HG]

   A Sketch the graph of $f(x) = x^3 - 9x^2 + 24x - 20$, showing all intercepts with the axes and turning points.

   B Find the equation of the tangent to $f(x)$ at $x = 4$.

   C Sketch the graph of $y = |f(x)|$ on a new set of axes, giving coordinates of the turning points and intercepts with the axes.

5. [IEB, Nov. 2004, HG] Solve: $18 \leq |x - 3|$

6. [IEB, Nov. 2005, HG] Solve for x: $25^{|1-2x|} = 5^4$

7. [IEB, Nov. 2005, HG] If $f(x) = |4 - x|$, find the value of $f'(3)$.

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