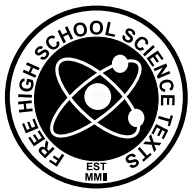

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The Free High School Science Texts: Textbooks for High School Students Studying the Sciences Mathematics Grades 10 - 12

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Contents

Chapter 19

Error Margins - Grade 11

We have seen that numbers are either rational or irrational and we have see how to round-off numbers. However, in a calculation that has many steps, it is best to leave the rounding off right until the end.

For example, if you were asked to write

$$3\sqrt{3} + \sqrt{12}$$

as a decimal number correct to two decimal places, there are two ways of doing this as described in Table 19.1.

Table 19.1: Two methods of writing $3\sqrt{3} + \sqrt{12}$ as a decimal number.

☺ **Method 1**		☹ **Method 2**	
$3\sqrt{3} + \sqrt{12}$	$= 3\sqrt{3} + \sqrt{4 \cdot 3}$	$3\sqrt{3} + \sqrt{12}$	$= 3 \times 1{,}73 + 3{,}46$
	$= 3\sqrt{3} + 2\sqrt{3}$		$= 5{,}19 + 3{,}46$
	$= 5\sqrt{3}$		$= 8{,}65$
	$= 5 \times 1{,}732050808\ldots$		
	$= 8{,}660254038\ldots$		
	$= 8{,}66$		

In the example we see that Method 1 gives 8,66 as an answer while Method 2 gives 8,65 as an answer. The answer of Method 1 is more accurate because the expression was simplified as much as possible before the answer was rounded-off.

In general, it is best to simplify any expression as much as possible, before using your calculator to work out the answer in decimal notation.

Important: Simplification and Accuracy

It is best to simplify all expressions as much as possible before rounding-off answers. This maintains the accuracy of your answer.

Worked Example 91: Simplification and Accuracy

Question: Calculate $\sqrt[3]{54} + \sqrt[3]{16}$. Write the answer to three decimal places.
Answer
Step 1 : Simplify the expression

$$\begin{aligned}\sqrt[3]{54}+\sqrt[3]{16} &= \sqrt[3]{27\cdot 2}+\sqrt[3]{8\cdot 2}\\ &= \sqrt[3]{27}\cdot\sqrt[3]{2}+\sqrt[3]{8}\cdot\sqrt[3]{2}\\ &= 3\sqrt[3]{2}+2\sqrt[3]{2}\\ &= 5\sqrt[3]{2}\\ &= 5\times 1{,}25992105\ldots\end{aligned}$$

Step 2 : Convert any irrational numbers to decimal numbers

$$\begin{aligned}5\sqrt[3]{2} &= 5\times 1{,}25992105\ldots\\ &= 6{,}299605249\ldots\\ &= 6{,}300\end{aligned}$$

Step 3 : Write the final answer to the required number of decimal places.

$6{,}299605249\ldots = 6{,}300$ to three decimal places

$\therefore\quad \sqrt[3]{54}+\sqrt[3]{16} = 6{,}300$ to three decimal places.

Worked Example 92: Simplification and Accuracy 2

Question: Calculate $\sqrt{x+1}+\frac{1}{3}\sqrt{(2x+2)-(x+1)}$ if $x = 3{,}6$. Write the answer to two decimal places.

Answer

Step 1 : Simplify the expression

$$\begin{aligned}\sqrt{x+1}+\frac{1}{3}\sqrt{(2x+2)-(x+1)} &= \sqrt{x+1}+\frac{1}{3}\sqrt{2x+2-x-1}\\ &= \sqrt{x+1}+\frac{1}{3}\sqrt{x+1}\\ &= \frac{4}{3}\sqrt{x+1}\end{aligned}$$

Step 2 : Substitute the value of x into the simplified expression

$$\begin{aligned}\frac{4}{3}\sqrt{x+1} &= \frac{4}{3}\sqrt{3{,}6+1}\\ &= \frac{4}{3}\sqrt{4{,}6}\\ &= 2{,}144761059\ldots\times 4\div 3\\ &= 2{,}859681412\ldots\end{aligned}$$

Step 3 : Write the final answer to the required number of decimal places.

$2{,}859681412\ldots = 2{,}86$ To two decimal places

$\therefore \sqrt{x+1}+\frac{1}{3}\sqrt{(2x+2)-(x+1)} = 2{,}86$ (to two decimal places) if $x = 3{,}6$.

Extension: Significant Figures

In a number, each non-zero digit is a significant figure. Zeroes are only counted if they are between two non-zero digits or are at the end of the decimal part. For example, the number 2000 has 1 significant figure (the 2), but $2000{,}0$ has 5 significant figures. Estimating a number works by removing significant figures from your number (starting from the right) until you have the desired number of significant figures, rounding as you go. For example $6{,}827$ has 4 significant figures, but if you wish to write it to 3 significant figures it would mean removing the 7 and rounding up, so it would be $6{,}83$. It is important to know when to estimate a number and when not to. It is usually good practise to only estimate numbers when it is absolutely necessary, and to instead use symbols to represent certain irrational numbers (such as π); approximating them only at the very end of a calculation. If it is necessary to approximate a number in the middle of a calculation, then it is often good enough to approximate to a few decimal places.

Appendix A

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