

FHSST Authors

The Free High School Science Texts: Textbooks for High School Students Studying the Sciences Physics Grades 10 - 12

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FHSST Core Team

Mark Horner; Samuel Halliday; Sarah Blyth; Rory Adams; Spencer Wheaton

FHSST Editors

Jaynie Padayachee ; Joanne Boulle ; Diana Mulcahy ; Annette Nell ; René Toerien ; Donovan Whitfield

FHSST Contributors

Rory Adams; Prashant Arora; Richard Baxter; Dr. Sarah Blyth; Sebastian Bodenstein; Graeme Broster; Richard Case; Brett Cocks; Tim Crombie; Dr. Anne Dabrowski; Laura Daniels ; Sean Dobbs ; Fernando Durrell ; Dr. Dan Dwyer ; Frans van Eeden ; Giovanni Franzoni ; Ingrid von Glehn ; Tamara von Glehn ; Lindsay Glesener ; Dr. Vanessa Godfrey ; Dr. Johan Gonzalez; Hemant Gopal; Umeshree Govender; Heather Gray; Lynn Greeff; Dr. Tom Gutierrez; Brooke Haag; Kate Hadley; Dr. Sam Halliday; Asheena Hanuman; Neil Hart; Nicholas Hatcher; Dr. Mark Horner; Robert Hovden; Mfandaidza Hove; Jennifer Hsieh; Clare Johnson; Luke Jordan; Tana Joseph; Dr. Jennifer Klay; Lara Kruger; Sihle Kubheka; Andrew Kubik; Dr. Marco van Leeuwen; Dr. Anton Machacek; Dr. Komal Maheshwari; Kosma von Maltitz; Nicole Masureik; John Mathew; JoEllen McBride; Nikolai Meures; Riana Meyer; Jenny Miller; Abdul Mirza; Asogan Moodaly; Jothi Moodley; Nolene Naidu; Tyrone Negus; Thomas O'Donnell; Dr. Markus Oldenburg; Dr. Jaynie Padayachee; Nicolette Pekeur; Sirika Pillay; Jacques Plaut; Andrea Prinsloo; Joseph Raimondo; Sanya Rajani ; Prof. Sergey Rakityansky ; Alastair Ramlakan ; Razvan Remsing ; Max Richter ; Sean Riddle; Evan Robinson; Dr. Andrew Rose; Bianca Ruddy; Katie Russell; Duncan Scott; Helen Seals; Ian Sherratt; Roger Sieloff; Bradley Smith; Greg Solomon; Mike Stringer; Shen Tian; Robert Torregrosa; Jimmy Tseng; Helen Waugh; Dr. Dawn Webber; Michelle Wen; Dr. Alexander Wetzler; Dr. Spencer Wheaton; Vivian White; Dr. Gerald Wigger; Harry Wiggins; Wendy Williams; Julie Wilson; Andrew Wood; Emma Wormauld; Sahal Yacoob; Jean Youssef

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Chapter 18

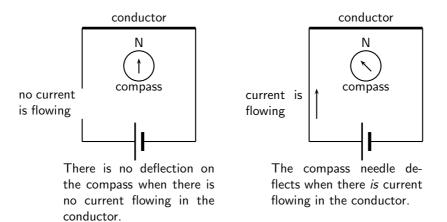
Electromagnetism - Grade 11

18.1 Introduction

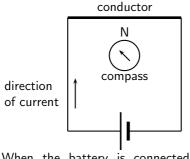
Electromagnetism is the science of the properties and relationship between electric currents and magnetism. An electric current creates a magnetic field and a moving magnetic field will create a flow of charge. This relationship between electricity and magnetism has resulted in the invention of many devices which are useful to humans.

18.2 Magnetic field associated with a current

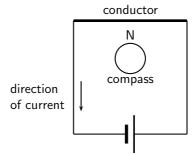
If you hold a compass near a wire through which current is flowing, the needle on the compass will be deflected.



Activity :: Case Study : Magnetic field near a current carrying conductor



When the battery is connected as shown, the compass needle is deflected to the left.



What do you think will happen if the direction of the current is reversed as shown?



The discovery of the relationship between magnetism and electricity was, like so many other scientific discoveries, stumbled upon almost by accident. The Danish physicist Hans Christian Oersted was lecturing one day in 1820 on the possibility of electricity and magnetism being related to one another, and in the process demonstrated it conclusively by experiment in front of his whole class. By passing an electric current through a metal wire suspended above a magnetic compass, Oersted was able to produce a definite motion of the compass needle in response to the current. What began as a guess at the start of the class session was confirmed as fact at the end. Needless to say, Oersted had to revise his lecture notes for future classes. His discovery paved the way for a whole new branch of science - electromagnetism.

The magnetic field produced by an electric current is always oriented perpendicular to the direction of the current flow. When we are drawing directions of magnetic fields and currents, we use the symbol \odot and \otimes . The symbol

 \odot

for an arrow that is coming out of the page and the symbol

 \otimes

for an arrow that is going into the page.

It is easy to remember the meanings of the symbols if you think of an arrow with a head and a tail.



When the arrow is coming out of the page, you see the head of the arrow (\odot) . When the arrow is going into the page, you see the tail of the arrow (\otimes) .

The direction of the magnetic field around the current carrying conductor is shown in Figure 18.1.

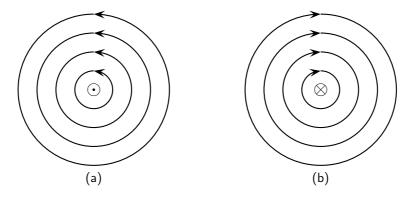


Figure 18.1: Magnetic field around a conductor when you look at the conductor from one end. (a) Current flows into the page and the magnetic field is counter clockwise. (b) Current flows out of the page and the magnetic field is clockwise.

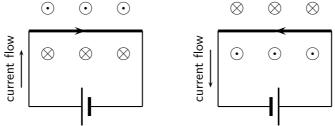
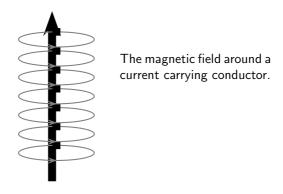


Figure 18.2: Magnetic fields around a conductor looking down on the conductor, for current in a conductor that is flowing to the right and to the left.

Activity :: Case Study : Direction of a magnetic field

Using the directions given in Figure 18.1 and Figure 18.2 and try to find a rule that easily tells you the direction of the magnetic field.

Hint: Use your fingers. Hold the wire in your hands and try to find a link between the direction of your thumb and the direction in which your fingers curl.



There is a simple method of showing the relationship between the direction of the current flowing in a conductor and the direction of the magnetic field around the same conductor. The method is called the *Right Hand Rule*. Simply stated, the right hand rule says that the magnetic flux lines produced by a current-carrying wire will be oriented the same direction as the curled fingers of a person's right hand (in the "hitchhiking" position), with the thumb pointing in the direction of the current flow.

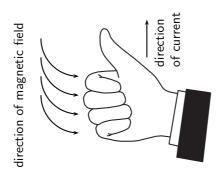
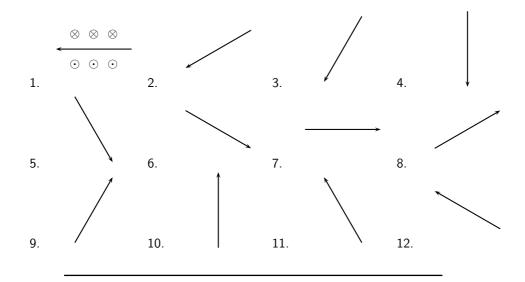


Figure 18.3: The Right Hand Rule.

Activity :: Case Study : The Right Hand Rule

Use the Right Hand Rule and draw in the directions of the magnetic field for the following conductors with the currents flowing in the directions shown by the arrow. The first problem has been completed for you.



Activity :: Experiment : Magnetic field around a current carrying conductor

Apparatus:

- 1. 1 9V battery with holder
- 2. 2 hookup wires with alligator clips
- 3. compass
- 4. stop watch

Method:

- 1. Connect your wires to the battery leaving one end unconnected so that the circuit is not closed.
- 2. One student should be in charge of limiting the current flow to 10 seconds. This is to preserve battery life as well as to prevent overheating of wires and battery contacts.
- 3. Place the compass close to the wire.
- 4. Close the circuit and observe what happens to the compass.

5. Reverse the polarity of the battery and close the circuit. Observe what happens to the compass.

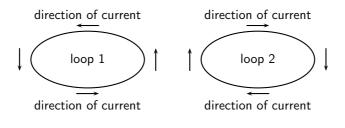
Conclusions:

Use your observations to answer the following questions:

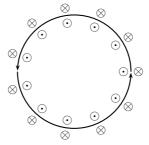
- 1. Does a current flowing in a wire generate a magnetic field?
- 2. Is the magnetic field present when the current is not flowing?
- 3. Does the direction of the magnetic field produced by a current in a wire depend on the direction of the current flow?
- 4. How does the direction of the current affect the magnetic field?

Activity :: Case Study : Magnetic field around a loop of conductor

Consider two loops of current carrying conductor that are placed in the plane of the page. Draw what you think the magnetic field would look like, by using the Right Hand Rule at different points of the two loops shown. Loop 1 has the current flowing in a counter-clockwise direction, while loop 2 has the current flowing in a clockwise direction.



If you make a loop of current carrying conductor, then the direction of the magnetic field is obtained by applying the Right Hand Rule to different points in the loop.



The directions of the magnetic field around a loop of current carrying conductor with the current flowing in a counter-clockwise direction is shown.

If we know add another loop then the magnetic field around each loop joins to create a stronger field. As more loops are added, the magnetic field gets a definite magnetic (north and south) polarity. Such a coil is more commonly known as a *solenoid*. The magnetic field pattern of a solenoid is similar to the magnetic field pattern of a bar magnet that you studied in Grade 10.

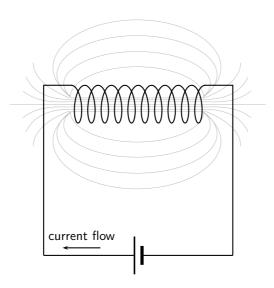


Figure 18.4: Magnetic field around a solenoid.

18.2.1 Real-world applications

Electromagnets

An *electromagnet* is a piece of wire intended to generate a magnetic field with the passage of electric current through it. Though all current-carrying conductors produce magnetic fields, an electromagnet is usually constructed in such a way as to maximize the strength of the magnetic field it produces for a special purpose. Electromagnets find frequent application in research, industry, medical, and consumer products.

As an electrically-controllable magnet, electromagnets find application in a wide variety of "electromechanical" devices: machines that effect mechanical force or motion through electrical power. Perhaps the most obvious example of such a machine is the *electric motor* which will be described in detail in Grade 12. Other examples of the use of electromagnets are electric bells, relays, loudspeakers and scrapyard cranes.

Activity :: Experiment : Electromagnets

Aim:

A magnetic field is created when an electric current flows through a wire. A single wire does not produce a strong magnetic field, but a coiled wire around an iron core does. We will investigate this behaviour.

Apparatus:

- 1. a battery and holder
- 2. a length of wire
- 3. a compass
- 4. a few nails
- 5. a few paper clips

Method:

- 1. Bend the wire into a series of coils before attaching it to the battery. Observe what happens to the deflection on the compass. Has the deflection of the compass grown stronger?
- 2. Repeat the experiment by changing the number and size of the coils in the wire. Observe what happens to the deflection on the compass.
- 3. Coil the wire around an iron nail and then attach the coil to the battery. Observe what happens to the deflection on the compass.

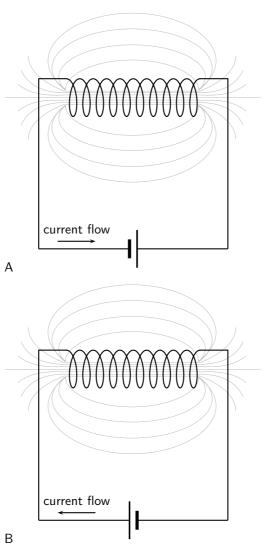
Conclusions:

- 1. Does the number of coils affect the strength of the magnetic field?
- 2. Does the iron nail increase or decrease the strength of the magnetic field?

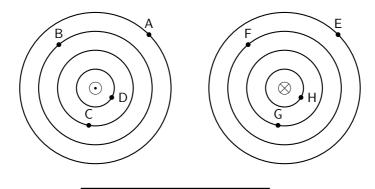
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Exercise: Magnetic Fields

- 1. Give evidence for the existence of a magnetic field near a current carrying wire.
- 2. Describe how you would use your right hand to determine the direction of a magnetic field around a current carrying conductor.
- 3. Use the right hand rule to determine the direction of the magnetic field for the following situations.



4. Use the Right Hand Rule to find the direction of the magnetic fields at each of the labelled points in the diagrams.



18.3 Current induced by a changing magnetic field

While Oersted's surprising discovery of electromagnetism paved the way for more practical applications of electricity, it was Michael Faraday who gave us the key to the practical generation of electricity: **electromagnetic induction**.

Faraday discovered that a voltage was generated across a length of wire while moving a magnet nearby, such that the distance between the two changed. This meant that the wire was exposed to a magnetic field flux of changing intensity. Furthermore, the voltage also depended on the orientation of the magnet; this is easily understood again in terms of the magnetic flux. The flux will be at its maximum as the magnet is aligned perpendicular to the wire. The magnitude of the changing flux and the voltage are linked. In fact, if the lines of flux are parallel to the wire, there will be no induced voltage.



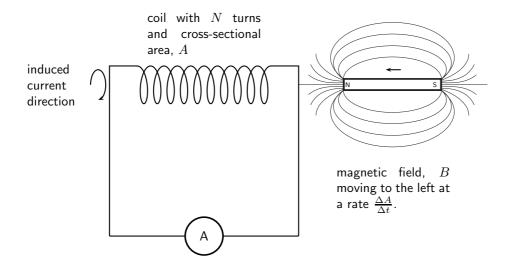
Definition: Faraday's Law

The emf, ϵ , produced around a loop of conductor is proportional to the rate of change of the magnetic flux, ϕ , through the area, A, of the loop. This can be stated mathematically as:

$$\epsilon = -N \frac{\Delta \phi}{\Delta t}$$

where $\phi = B \cdot A$ and B is the strength of the magnetic field.

Faraday's Law relates induced emf to the rate of change of flux, which is the product of the magnetic field and the cross-sectional area the field lines pass through.



When the north pole of a magnet is pushed into a solenoid, the flux in the solenoid increases so the induced current will have an associated magnetic field pointing out of the solenoid (opposite to the magnet's field). When the north pole is pulled out, the flux decreases, so the induced current will have an associated magnetic field pointing into the solenoid (same direction as the magnet's field) to try to oppose the change. The directions of currents and associated magnetic fields can all be found using only the Right Hand Rule. When the fingers of the right hand are pointed in the direction of the current, the thumb points in the direction of the magnetic field. When the thumb is pointed in the direction of the magnetic field, the fingers point in the direction of the current.

Important: An easy way to create a magnetic field of changing intensity is to move a permanent magnet next to a wire or coil of wire. The magnetic field must increase or decrease in intensity *perpendicular* to the wire (so that the lines of flux "cut across" the conductor), or else no voltage will be induced.

Important: Finding the direction of the induced current

The induced current generates a magnetic field. The induced magnetic field is in a direction that cancels out the magnetic field in which the conductor is moving. So, you can use the Right Hand Rule to find the direction of the induced current by remembering that the induced magnetic field is opposite in direction to the magnetic field causing the change.

Electromganetic induction is put into practical use in the construction of electrical generators, which use mechanical power to move a magnetic field past coils of wire to generate voltage. However, this is by no means the only practical use for this principle.

If we recall that the magnetic field produced by a current-carrying wire was always perpendicular to that wire, and that the flux intensity of that magnetic field varied with the amount of current through it, we can see that a wire is capable of inducing a voltage along its own length simply due to a change in current through it. This effect is called self-induction. Self-induction is when a changing magnetic field is produced by changes in current through a wire inducing voltage along the length of that same wire.

If the magnetic field flux is enhanced by bending the wire into the shape of a coil, and/or wrapping that coil around a material of high permeability, this effect of self-induced voltage will be more intense. A device constructed to take advantage of this effect is called an *inductor*, and will be discussed in greater detail in the next chapter.



Extension: Lenz's Law

The induced current will create a magnetic field that opposes the change in the magnetic flux.



Worked Example 121: Faraday's Law

Question: Consider a flat square coil with 5 turns. The coil is 0.50 m on each side, and has a magnetic field of 0.5 T passing through it. The plane of the coil is perpendicular to the magnetic field: the field points out of the page. Use Faraday's Law to calculate the induced emf if the magnetic field is increases uniformly from 0.5 T to 1 T in 10 s. Determine the direction of the induced current.

Answer

Step 1: Identify what is required

We are required to use Faraday's Law to calculate the induced emf.

Step 2: Write Faraday's Law

$$\epsilon = -N \frac{\Delta \phi}{\Delta t}$$

$$421$$

Step 3 : Solve Problem

$$\epsilon = -N \frac{\Delta \phi}{\Delta t}$$

$$= -N \frac{\phi_f - \phi_i}{\Delta t}$$

$$= -N \frac{B_f \cdot A - B_i \cdot A}{\Delta t}$$

$$= -N \frac{A(B_f - B_i)}{\Delta t}$$

$$= -(5) \frac{(0.5)^2 (1 - 0.5)}{10}$$

$$= 0.0625 \text{ V}$$

18.3.1 Real-life applications

The following devices use Faraday's Law in their operation.

- induction stoves
- tape players
- metal detectors
- transformers

Activity :: Research Project : Real-life applications of Faraday's Law

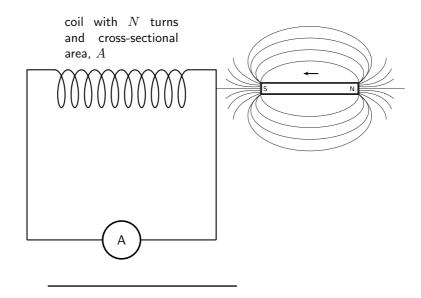
Choose one of the following devices and do some research on the internet or in a library how your device works. You will need to refer to Faraday's Law in your explanation.

- induction stoves
- tape players
- metal detectors
- transformers



Exercise: Faraday's Law

- 1. State Faraday's Law in words and write down a mathematical relationship.
- 2. Describe what happens when a bar magnet is pushed into or pulled out of a solenoid connected to an ammeter. Draw pictures to support your description.
- 3. Use the right hand rule to determine the direction of the induced current in the solenoid below.



18.4 Transformers

One of the real-world applications of Faraday's Law is in a transformer.

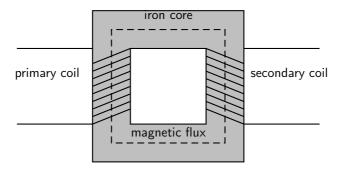
Eskom generates electricity at around 22 000 V. When you plug in a toaster, the mains voltage is 220 V. A transformer is used to *step-down* the high voltage to the lower voltage that is used as mains voltage.



Definition: Transformer

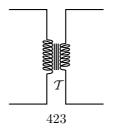
A transformer is an electrical device that uses the principle of induction between the primary coil and the secondary coil to either step-up or step-down voltage.

The essential features of a transformer are two coils of wire, called the primary coil and the secondary coil, which are wound around different sections of the same iron core.



When an alternating voltage is applied to the primary coil it creates an alternating current in that coil, which induces an alternating magnetic field in the iron core. This changing magnetic field induces an emf, which creates a current in the secondary coil.

The circuit symbol for a transformer is:



A very useful property of transformers is the ability to transform voltage and current levels according to a simple ratio, determined by the ratio of input and output coil turns. We can derive a mathematical relationship by using Faraday's law.

Assume that an alternating voltage V_p is applied to the primary coil (which has N_p turns) of a transformer. The current that results from this voltage generates a magnetic flux ϕ_p . We can then describe the emf in the primary coil by:

$$V_p = N_p \frac{\Delta \phi_p}{\Delta t}$$

Similarly, for the secondary coil,

$$V_s = N_s \frac{\Delta \phi_s}{\Delta t}$$

If we assume that the primary and secondary windings are perfectly coupled, then:

$$\phi_p = \phi_s$$

which means that:

$$\frac{V_p}{V_s} = \frac{N_p}{N_s}$$



Worked Example 122: Transformer specifications

Question: Calculate the voltage on the secondary coil if the voltage on the primary coil is 120 V and the ratio of primary windings to secondary windings is 10:1.

Answer

 $\label{eq:Step 1: Determine how to approach the problem} \textbf{Step 1: Determine how to approach the problem}$

Use

$$\frac{V_p}{V_r} = \frac{N_p}{N_r}$$

with

- $V_p = 120$
- $\bullet \ \frac{N_p}{N_s} = \frac{10}{1}$

Step 2 : Rearrange equation to solve for $\ensuremath{V_s}$

$$\frac{V_p}{V_s} = \frac{N_p}{N_s}$$

$$\frac{1}{V_s} = \frac{N_p}{N_s} \frac{1}{V_p}$$

$$\therefore V_s = \frac{1}{\frac{N_p}{N_s}} V_p$$

Step 3 : Substitute values and solve for V_s

$$V_s = \frac{1}{\frac{N_p}{N_s}} V_p$$
$$= \frac{1}{\frac{10}{1}} 120$$
$$= 12 \text{ V}$$

A transformer designed to output more voltage than it takes in across the input coil is called a *step-up* transformer. A step-up transformer has more windings on the secondary coil than on the primary coil. This means that:

$$N_s > N_p$$

Similarly, a transformer designed to output less than it takes in across the input coil is called a *step-down* transformer. A step-down transformer has more windings on the primary coil than on the primary coil. This means that:

$$N_p > N_s$$

We use a step-up transformer to increase the voltage from the primary coil to the secondary coil. It is used at power stations to increase the voltage for the transmission lines. A step-down transformer decreases the voltage from the primary coil to the secondary coil. It is particularly used to decrease the voltage from the transmission lines to a voltage which can be used in factories and in homes.

Transformer technology has made long-range electric power distribution practical. Without the ability to efficiently step voltage up and down, it would be cost-prohibitive to construct power systems for anything but close-range (within a few kilometres) use.

As useful as transformers are, they only work with AC, not DC. This is because the phenomenon of mutual inductance relies on *changing* magnetic fields, and direct current (DC) can only produce steady magnetic fields, transformers simply will not work with direct current.

Of course, direct current may be interrupted (pulsed) through the primary winding of a transformer to create a changing magnetic field (as is done in automotive ignition systems to produce high-voltage spark plug power from a low-voltage DC battery), but pulsed DC is not that different from AC. Perhaps more than any other reason, this is why AC finds such widespread application in power systems.

18.4.1 Real-world applications

Transformers are very important in the supply of electricity nationally. In order to reduce energy losses due to heating, electrical energy is transported from power stations along power lines at high voltage and low current. Transformers are used to step the voltage up from the power station to the power lines, and step it down from the power lines to buildings where it is needed.

?

Exercise: Transformers

- 1. Draw a sketch of the main features of a transformer
- 2. Use Faraday's Law to explain how a transformer works in words and pictures.
- 3. Use the equation for Faraday's Law to derive an expression involving the ratio between the voltages and number of windings in the primary and secondary coils
- 4. If we have $N_p=100$ and $N_s=50$ and we connect the primary winding to a 230 V, 50Hz supply then calculate the voltage on the secondary winding.
- 5. State the difference between a step-up and a step-down transformer in both structure and function.
- 6. Give an example of the use of transformers.

18.5 Motion of a charged particle in a magnetic field

When a charged particle moves through a magnetic field it experiences a force. For a particle that is moving at right angles to the magnetic field, the force is given by:

$$F = qvB$$

where q is the charge on the particle, v is the velocity of the particle and B is the magnetic field through which the particle is moving.





Worked Example 123: Charged particle moving in a magnetic field

Question: An electron travels at $150 \mathrm{m.s^{-1}}$ at right angles to a magnetic field of 80 000 T. What force is exerted on the electron?

Answer

Step 1 : Determine what is required

We are required to determine the force on a moving charge in a magnetic field

Step 2: Determine how to approach the problem

We can use the formula:

$$F = qvB$$

Step 3: Determine what is given

We are given

- $q = 1.6 \times 10^{-19} \text{C}$ (The charge on an electron)
- $v = 150 \text{m.s}^{-1}$
- B = 80~000T

Step 4: Determine the force

$$F = qvB$$
= $(1.6 \times 10^{-19} \text{C})(150 \text{m.s}^{-1})(80\ 000 \text{T})$
= $1.92 \times 10^{-12} \text{N}$



Important: The direction of the force exerted on a charged particle moving through a magnetic field is determined by using the Right Hand Rule.

Point your fingers in the direction of the velocity of the charge and turn them (as if turning a screwdriver) towards the direction of the magnetic field. Your thumb will point in the direction of the force. If the charge is negative, the direction of the force will be opposite to the direction of your thumb.

18.5.1 Real-world applications

The following devices use the movement of charge in a magnetic field

- televisions
- oscilloscope

Activity :: Research Project : Real-life applications of charges moving in a magnetic field

Choose one of the following devices and do some research on the internet or in a library how your device works.

- oscilloscope
- television

?

Exercise: Lorentz Force

- 1. What happens to a charged particle when it moves through a magnetic field?
- 2. Explain how you would use the Right Hand Rule to determine the direction of the force experienced by a charged particle as it moves in a magnetic field.
- 3. Explain how the force exerted on a charged particle moving through a magnetic field is used in a television.

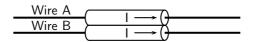
18.6 Summary

- 1. Electromagnetism is the study of the properties and relationship between electric current and magnetism.
- 2. A current carrying conductor will produce a magnetic field around the conductor.
- 3. The direction of the magnetic field is found by using the Right Hand Rule.
- 4. Electromagnets are temporary magnets formed by current-carrying conductors.
- 5. Electromagnetic induction occurs when a moving magnetic field induces a voltage in a current-carrying conductor.
- 6. Transformers use electromagnetic induction to alter the voltage.
- 7. A charged particle will experience a force in a magnetic field.

18.7 End of chapter exercises

- 1. State the Right Hand Rule.
- 2. What did Hans Oersted discover about the relationship between electricity and magnetism?
- 3. List two uses of electromagnetism.
- 4. Draw a labelled diagram of an electromagnet and show the poles of the electromagnet on your sketch.
- 5. Transformers are useful electrical devices.
 - A What is a transformer?
 - B Draw a sketch of a step-down transformer?
 - C What is the difference between a step-down and step-up transformer?

- D When would you use a step-up transformer?
- 6. Calculate the voltage on the secondary coil of a transformer if the voltage on the primary coil is 22 000 V and the ratio of secondary windings to secondary windings is 500:1.
- 7. You find a transformer with 1000 windings on the primary coil and 200 windinds on the secondary coil.
 - A What type of transformer is it?
 - B What will be the voltage on the secondary coil if the voltage on the primary coil is 400 V?
- IEB 2005/11 HG An electric cable consists of two long straight parallel wires separated by plastic insulating material. Each wire carries a current I in the same direction (as shown in the diagram below).



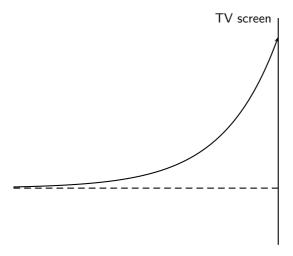
Which of the following is true concerning the force of Wire A on Wire B?

	Direction of Force	Origin of Force
(a)	towards A (attraction)	electrostatic force between opposite charges
(b)	towards B (repulsion)	electrostatic force between opposite charges
(c)	towards A (attraction)	magnetic force on current-carrying conductor
(d)	towards B (repulsion)	magnetic force on current-carrying conductor

IEB 2005/11 HG1 Force of parallel current-carrying conductors

Two long straight parallel current-carrying conductors placed $1\ m$ apart from each other in a vacuum each carry a current of $1\ A$ in the same direction.

- A What is the magnitude of the force of 1 m of one conductor on the other?
- B How does the force compare with that in the previous question when the current in one of the conductors is halved, and their distance of separation is halved?
- IEB 2005/11 HG An electron moving horizontally in a TV tube enters a region where there is a uniform magnetic field. This causes the electron to move along the path (shown by the solid line) because the magnetic field exerts a constant force on it. What is the direction of this magnetic field?



- A upwards (towards the top of the page)
- B downwards (towards the bottom of the page)
- C into the page
- D out of the page

Appendix A

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