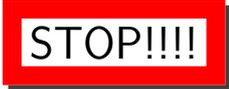FHSST Authors

The Free High School Science Texts: Textbooks for High School Students Studying the Sciences Physics Grades 10 - 12

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Chapter 31

Optical Phenomena and Properties of Matter - Grade 12

31.1 Introduction

For centuries physicists argued about whether light was a particle or a wave. It was assumed that light could only be one or the other, but not both.

In earlier chapters on waves (Chapters 6, 24, 25, 26) and optics (Chapters 7 and 13), you studied how light or other electromagnetic radiation propagates like a wave. The wave nature of light of was demonstrated by the propagation of light in examples such as diffraction, interference, and polarisation of light.

You also saw in Chapter 30 on electromagnetic radiation how light sometimes behaves as a particle. This chapter looks at evidence supporting the *particle model of light*. The idea that light can have both wave and particle properties was one of the most important discoveries of the twentieth century.

31.2 The transmission and scattering of light

31.2.1 Energy levels of an electron

We have seen that the electrons in an atom have different energy levels. When the electron receives enough energy, it can jump up to a higher energy level. This is called *'exciting'* the electron. When the electron in a high energy level sheds some energy, it drops to a lower energy level.

We have also seen that the energy associated with light at a specific wavelength is given by:

$$E = \frac{hc}{\lambda}.$$

In the particle model of light, this means that each packet of light (photon) at a wavelength λ has energy:

$$E = \frac{hc}{\lambda}.$$

For the electron to receive energy, it *absorbs* a photon and gets its energy. When an electron loses energy to drop to a lower level, it *emits* (gives off) a photon with that energy.

31.2.2   Interaction of light with metals

When light encounters or passes through a material, the photons of the light interact with the atoms or molecules of the material. Depending on the strength of the interactions and how often they happen, the light will pass through the material or be scattered in some other direction.

Each wavelength of light relates to a particular energy, and the closer that energy is to the energy difference between two of the levels of the atom, the likelier the photon is to interact with the atom.

When visible or ultraviolet (UV) radiation shines on a metal, the photons are *absorbed* by the electrons in the metal. The electrons are then excited up to a higher energy level. When an electron returns to a lower energy level, another photon is emitted. This is how light is **reflected** off a metal surface.

In previous chapters, you have studied geometrical optics, which tells us what happens to rays of light when they are reflected off a surface or refracted through a lens. That tells us what happens to light rays, made up of many photons, on a large scale. If you look at a smaller level, i.e. on a microscopic scale, then reflection and refraction happen by all the photons interacting with the atoms of the lens or mirror. The photons get absorbed and re-emitted many times before emerging as the finals rays of light that we see.

Scattering of light is responsible for many effects in everyday life. We see that certain materials are red or blue, for example, since they contain materials that have energy level differences that correspond to the energies of the photons that make up red or blue light. These materials then reflect the red or blue light and absorb the other wavelengths in the visible spectrum. White objects reflect photons of all wavelengths in the visual spectrum, while black objects absorb these photons.

Interesting Fact

Because a truly black object absorbs all the visual wavelengths of light, and does not re-emit photons at visual wavelengths, we can say that 'black' is not a colour itself, but rather a *lack of* colour! Also, since black objects absorb visual light, they heat up more than objects of other colours which reflect light at certain wavelengths.

Activity :: Investigation : Reflection and absorption
Aim:
Investigate the interaction of light with differently coloured metal objects
Apparatus:
Find some differently coloured metal objects (at least 5) which will not be damaged if left in the sun for 15 minutes. Make sure to include at least one white item and one black item.
Method:
At the start of your lesson set out your objects in direct sunlight. Leave them there for around 15 minutes.

Alternatively, if it is a sunny day, you can use your teachers' cars for this experiment - as long as there are some cars of different colours and they have been standing in direct sunlight for the same length of time!

After 15 minutes is up, touch each of the items/cars (be careful not to burn yourself!) and compare their temperatures (rate them from 1 to 5 with 1 being cold and 5 being very hot) in a table such as the example table below:
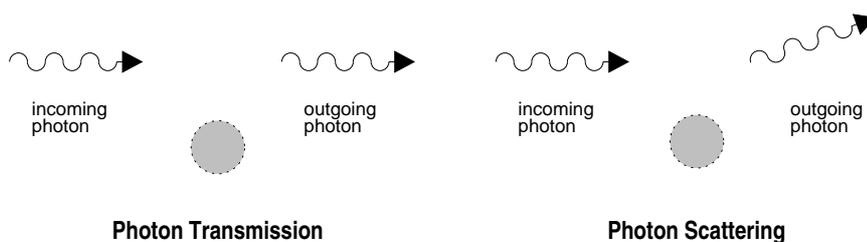
Figure 31.1: Diagrams of photon transmission (left) and scattering (right).

Object	Colour	Temperature rating
e.g. car 1	e.g. red	e.g. 3

Questions:

1. Which object was the hottest and what was it made of?
2. Which object was the coolest and what was it made of?
3. How did the temperatures of the black and white objects compare to each other? (which one was hotter and which was cooler?)
4. Try to explain the reasons for the different temperatures of the objects with respect to their colours and the materials of which they are made.

Metals generally reflect most wavelengths of visible light, but they will reflect the light in a certain direction, given by the laws of reflection in geometrical optics. This is different to most materials, like wood or fabric, which reflect light in all directions. Metals have this property since they have electrons that are not bound to atoms and can move freely through the metal. This is unlike most other materials that have their electrons bound closely to the atoms. These free electrons in metals can then absorb and reflect photons of a wide range of energies.

Ultraviolet light (which has shorter wavelengths than visible light) will pass through some substances, such as many plastics, because they do not have the right energy levels to absorb it and re-emit it. *X-rays* (also short wavelengths) will also pass through most materials, since the energies of X-rays correspond to the energy levels of atomic nuclei. Such nuclei are much smaller than atoms, so it is much less likely for an X-ray to hit a nucleus instead of the whole atom.

Most materials will absorb *infrared radiation* (longer wavelengths than visible light), since the energies of that radiation often correspond to rotational or vibrational energy levels of molecules.

31.2.3 Why is the sky blue?

The sun emits light in many different wavelengths, including all of the visible wavelengths. Light which is made up of all the visible wavelengths appears white. So what causes the sky to look blue?

The air is not just full of nitrogen and oxygen gases. It is also full of tiny dust grains. The light from the sun scatters off these many dust grains.

The chance that the light will scatter off one of these dust grains is *bigger* for *shorter* wavelengths. The *short wavelength* **blue light** is therefore scattered much more than the other colors. At noon, when the light from the sun is coming straight down (see the picture below), the scattered blue light reaches your eyes from all directions and so the sky appears blue. The other wavelengths do not get scattered much and therefore miss your line of sight. At sunrise

or sunset, the direction of the light coming from the sun is now straight towards your eyes (see picture below). Therefore the scattered blue light can't be seen because it is scattered out of your line of sight. The redder colours (oranges and reds) can now be seen because they are not scattered as much and still fall in your line of sight.

At noon, sun is overhead

sun emits radiation
of all wavelengths

blue light scattered
by large angle
into the eye

red light scattered little
and misses the eye

At sunrise/sunset, sun is at horizon

blue light scattered
by large angle
away from the eye

red light scattered
into the eye

?
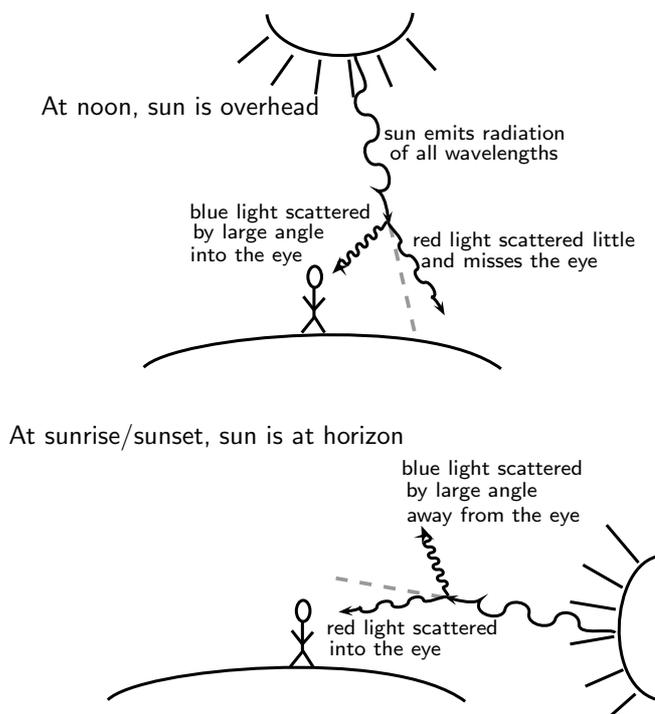
Exercise: Transmission and scattering of light

1. Explain how visible light is reflected from metals.

2. Explain why the sky is blue.

31.3   The photoelectric effect

Around the turn of the twentieth century, it was observed by a number of physicists (including Hertz, Thomson and Von Lenard) that when light was shone on a metal, electrons were emitted by the metal. This is called the photoelectric effect. (*photo-* for light, *electric-* for the electron.)

Definition: The photoelectric effect
The photoelectric effect is the process whereby an electron is emitted by a metal when light shines on it.

At that time, light was thought to be purely a wave. Therefore, physicists thought that if a more intense (i.e. brighter) light was shone on a metal, then the electrons would be knocked out with greater kinetic energies than if a faint light was shone on them. However, Von Lenard

observed that this did not happen at all. The *intensity* of the light made *no difference* to the kinetic energy of the emitted electrons! Also, it was observed that the electrons were emitted immediately when light was shone on the metal - there was no time delay.

Einstein solved this problem by proposing that light is made up of packets of energy called *quanta* (now called photons) which interacted with the electrons in the metal like particles instead of waves. Each incident photon would transfer all its energy to one electron in the metal. For a specific colour of light (i.e. a certain wavelength or frequency), the energy of the photons is given by $E = hf = hc/\lambda$, where h is Planck's constant. The energy needed to knock an electron out of the metal is called the **work function** (symbol ϕ) of the metal. Therefore, the amount of energy left over as the kinetic energy (E_k) of the emitted electron would be the difference between the incoming photon's energy and the energy needed to knock out the electron (work function of the metal):

$$E_k \quad = \quad hf - \phi$$

Increasing the intensity of the light (i.e. making it brighter) did not change the wavelength of the light and therefore the electrons would be emitted with *the same* kinetic energy as before! This solved the paradox and showed that light has *both* a **wave nature** and a **particle nature**. Einstein won the Nobel prize for this quantum theory and his explanation of the photoelectric effect.

Increasing the intensity of the light actually means increasing the *number* of incident photons. Therefore, since each photon only gives energy to one electron, more incident photons means*more* electrons would be knocked out of the metal, but their kinetic energies would be *the same* as before.
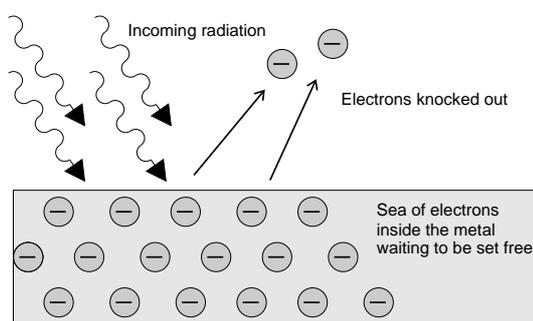
Figure 31.2: The photoelectric effect: Incoming photons on the left hit the electrons inside the metal surface. The electrons absorb the energy from the photons, and are ejected from the metal surface.

Interesting Fact
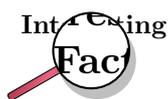
The photoelectric effect was first observed in the experiments of Heinrich Hertz in 1887. In 1899 J.J. Thomson proved that it was electrons that were emitted. The photoelectric effect was theoretically explained by Albert Einstein in 1905.

The discovery and understanding of the photoelectric effect was one of the major breakthroughs in science in the twentieth century as it provided concrete evidence of the particle nature of light. It overturned previously held views that light was composed purely of a continuous transverse wave. On the one hand, the wave nature is a good description of phenomena such as diffraction and interference for light, and on the other hand, the

photoelectric effect demonstrates the particle nature of light. This is now known as the 'dual-nature' of light. (dual means two)

While solving problems we need to decide for ourselves whether we should consider the wave property or the particle property of light. For example, when dealing with interference and diffraction, light should be treated as a wave, whereas when dealing with photoelectric effect we consider the particle nature.

31.3.1 Applications of the photoelectric effect

We have learnt that a metal contains electrons that are free to move between the valence and conduction bands. When a photon strikes the surface of a metal, it gives *all* its energy to one electron in the metal.

- If the photon energy is *equal* to the energy between two energy levels then the electron is excited to the higher energy level.

- If the photon energy is *greater* than or *equal to* the work function (energy needed to escape from the metal), then the electron is emitted from the surface of the metal (the photoelectric effect).

The work function is different for different elements. The smaller the work function, the easier it is for electrons to be emitted from the metal. Metals with low work functions make good conductors. This is because the electrons are attached less strongly to their surroundings and can move more easily through these materials. This reduces the resistance of the material to the flow of current i.e. it conducts well. Table 31.1 shows the work functions for a range of elements.

Element	Work Function (J)
Aluminium	$6{,}9 \times 10^{-19}$
Beryllium	$8{,}0 \times 10^{-19}$
Calcium	$4{,}6 \times 10^{-19}$
Copper	$7{,}5 \times 10^{-19}$
Gold	$8{,}2 \times 10^{-19}$
Lead	$6{,}9 \times 10^{-19}$
Silicon	$1{,}8 \times 10^{-19}$
Silver	$6{,}9 \times 10^{-19}$
Sodium	$3{,}7 \times 10^{-19}$

Table 31.1: Work functions of selected elements determined from the photoelectric effect. (From the *Handbook of Chemistry and Physics.*)

Interesting Fact

The **electron volt** is the kinetic energy gained by an electron passing through a potential difference of one volt (1 V). A **volt** is not a measure of energy, but the **electron volt** is a unit of energy. When you connect a 1.5 Volt battery to a circuit, you can give 1.5 eV of energy to every electron.

Worked Example 189:   The photoelectric effect - I

Question: Ultraviolet radiation with a wavelength of 250 nm is incident on a silver foil (work function $\phi = 6,9 \times 10^{-19}$). What is the maximum kinetic energy of the emitted electrons?

Answer

Step 1 : Determine what is required and how to approach the problem

We need to determine the maximum kinetic energy of an electron ejected from a silver foil by ultraviolet radiation.

The photoelectric effect tells us that:

$$\begin{aligned} E_k &= E_{photon} - \phi \\ E_k &= h\frac{c}{\lambda} - \phi \end{aligned}$$

We also have:

Work function of silver: $\phi = 6,9 \times 10^{-19}$ J

UV radiation wavelength $= 250$ nm $= 250 \times 10^{-9}$ m

Planck's constant: $h = 6,63 \times 10^{-34}$ m^{2}kgs^{-1}

speed of light: $c = 3 \times 10^8$ ms^{-1}

Step 2 : Solve the problem

$$\begin{aligned} E_k &= \frac{hc}{\lambda} - \phi \\ &= [6,63 \times 10^{-34} \times \frac{3 \times 10^8}{250 \times 10^{-9}}] - 6,9 \times 10^{-19} \\ &= 1,06 \times 10^{-19} \text{ J} \end{aligned}$$

The maximum kinetic energy of the emitted electron will be $1,06 \times 10^{-19}$ J.

Worked Example 190:   The photoelectric effect - II

Question: If we were to shine the same ultraviolet radiation ($f = 1,2 \times 10^{15}$ Hz), on a gold foil (work function $= 8,2 \times 10^{-19}$ J), would any electrons be emitted from the surface of the gold foil?

Answer

For the electrons to be emitted from the surface, the energy of each photon needs to be *greater* than the work function of the material.

Step 1 : Calculate the energy of the incident photons

$$\begin{aligned} E_{photon} &= hf \\ &= 6,63 \times 10^{-34} \times 1,2 \times 10^{15} \\ &= 7,96 \times 10^{-19} \text{ J} \end{aligned}$$

Therefore each photon of ultraviolet light has an energy of $7,96 \times 10^{-19}$ J.

Step 2 : Write down the work function for gold.

$$\phi_{gold} = 8,2 \times 10^{-19} \text{ J}$$

Step 3 : Is the energy of the photons greater or smaller than the work function?

$$7{,}96 \times 10^{-19} \text{ J} \quad < \quad 8{,}2 \times 10^{-19} \text{ J}$$
$$E_{photons} \quad < \quad \phi_{gold}$$

The energy of each photon is less than the work function of gold. Therefore, the photons do not have enough energy to knock electrons out of the gold. No electrons would be emitted from the gold foil.

Extension: Units of energy
When dealing with calculations at a small scale (like at the level of electrons) it is more convenient to use different units for energy rather than the Joule (J). We define a unit called the electron-Volt (eV) as the kinetic energy gained by an electron passing through a potential difference of one volt.

$$E = q \times V$$

where q is the charge of the electron and V is the potential difference applied. The charge of 1 electron is $1{,}6 \times 10^{-19}$ C, so 1 eV is calculated to be:

$$1 \text{ eV} = (1{,}610^{-19} \text{ C} \times 1 \text{ V}) = 1{,}6 \times 10^{-19} \text{ J}$$

You can see that $1{,}6 \times 10^{-19}$ J is a very small amount of energy and so using electron-Volts (eV) at this level is easier.
Hence, $1 \text{eV} = 1.6 \times 10^{-}19$ J which means that $1 \text{ J} = 6.241 \times 10^{18}$ eV

31.3.2   Real-life applications

Solar Cells

The photo-electric effect may seem like a very easy way to produce electricity from the sun. This is why people choose to make solar panels out of materials like silicon, to generate electricity. In real-life however, the amount of electricity generated is less than expected. This is because not every photon knocks out an electron. Other processes such as reflection or scattering also happen. This means that only a fraction $\approx 10\%$ (depends on the material) of the photons produce photoelectrons. This drop in efficiency results in a lower measured current. Much work is being done in industry to improve this efficiency so that the panels can generate as high a current as possible, and create as much electricity as possible form the sun. But even these smaller electrical currents are useful in applications like solar-powered calculators.

Exercise: The photoelectric effect

1. Describe the photoelectric effect.

2. List two reasons why the observation of the photoelectric effect was significant.

3. Refer to Table 31.1: If I shine ultraviolet light with a wavelength of 288 nm onto some aluminium foil, what would be the kinetic energy of the emitted electrons?

4. I shine a light of an unknown wavelength onto some silver foil. The light has only enough energy to eject electrons from the silver foil but not enough to give them kinetic energy. (Refer to Table 31.1 when answering the questions below:)

    A If I shine the same light onto some copper foil, would electrons be ejected?

    B If I shine the same light onto some silicon, would electrons be ejected?

    C If I increase the intensity of the light shining on the silver foil, what happens?

    D If I increase the frequency of the light shining on the silver foil, what happens?

31.4 Emission and absorption spectra

31.4.1 Emission Spectra

You have learnt previously about the structure of an atom. The electrons surrounding the atomic nucleus are arranged in a series of levels of increasing energy. Each element has its own distinct set of energy levels. This arrangement of energy levels serves as the atom's unique fingerprint.

In the early 1900s, scientists found that a liquid or solid heated to high temperatures would give off a broad range of colours of light. However, a gas heated to similar temperatures would emit light only at certain specific colours (wavelengths). The reason for this observation was not understood at the time.
Scientists studied this effect using a discharge tube.
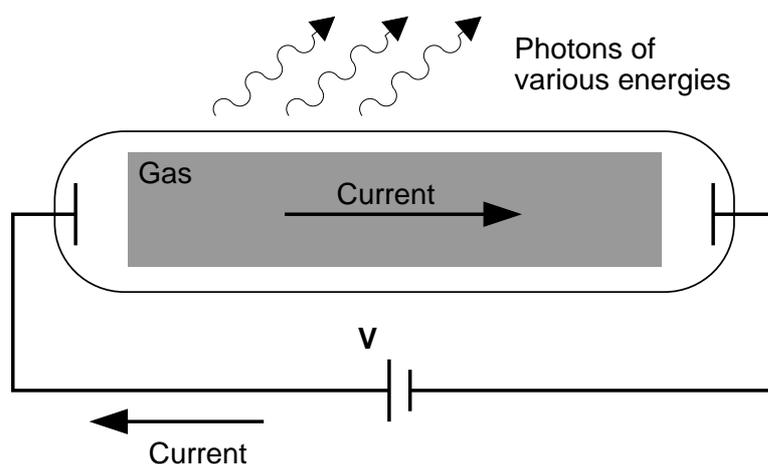
Figure 31.3: Diagram of a discharge tube. The tube is filled with a gas. When a high enough voltage is applied to both ends of the tube, the gas ionises and acts like a conductor, allowing a current to flow through the circuit. The current excites the atoms of the ionised gas. When the atoms fall back to their ground state, they emit photons to carry off the excess energy.

A discharge tube (Figure 31.3) is a glass gas-filled tube with a metal plate at both ends. If a large enough voltage difference is applied between the two metal plates, the gas atoms inside the tube will absorb enough energy to make some of their electrons come off i.e. the gas atoms are ionised. These electrons start moving through the gas and create a current, which raises some electrons in other atoms to higher energy levels. Then as the electrons in the atoms fall

back down, they emit electromagnetic radiation. The amount of light emitted at different wavelengths, called the **emission spectrum**, is shown for a discharge tube filled with hydrogen gas in figure 31.4 below. Only certain wavelengths (i.e. colours) of light are seen as shown by the thick black lines in the picture.
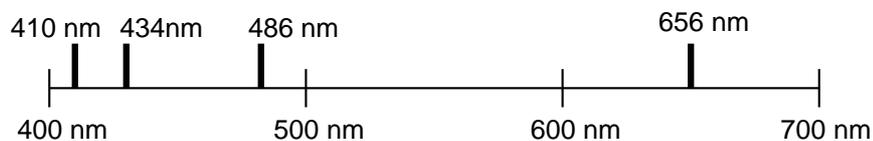
Figure 31.4: Diagram of the emission spectrum of hydrogen in the visible spectrum. Four lines are visible, and are labeled with their wavelengths. The three lines in the 400–500 nm range are in the blue part of the spectrum, while the higher line (656 nm) is in the red/orange part.

Eventually, scientists realized that these lines come from photons of a specific energy, emitted by electrons making transitions between specific energy levels of the atom. Figure **??** shows an example of this happening. When an electron in an atom falls from a higher energy level to a lower energy level, it emits a photon to carry off the extra energy. This photon's energy is equal to the energy difference between the two energy levels. As we previously discussed, the frequency of a photon is related to its energy through the equation $E = hf$. Since a specific photon frequency (or wavelength) gives us a specific colour, we can see how each coloured line is associated with a specific transition.
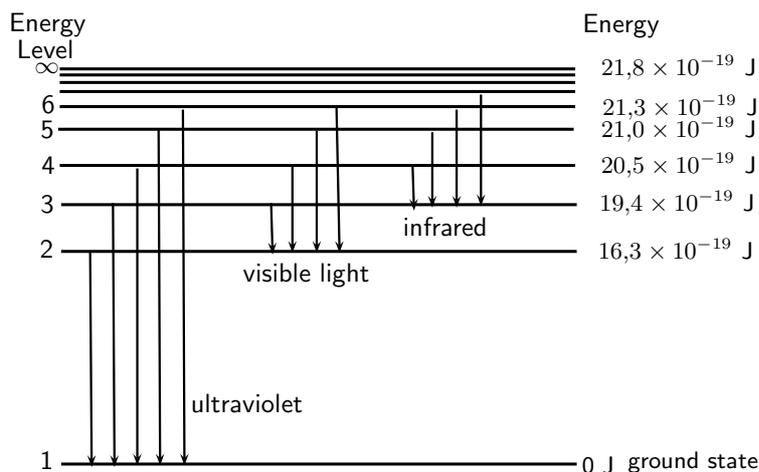
Figure 31.5: In this diagram are shown some of the electron energy levels for the hydrogen atom. The arrows show the electron transitions from higher energy levels to lower energy levels. The energies of the emitted photons are the same as the energy difference between two energy levels. You can think of absorption as the opposite process. The arrows would point upwards and the electrons would jump up to higher levels when they absorp a photon of the right energy.

Visible light is not the only kind of electromagnetic radiation emitted. More energetic or less energetic transitions can produce ultraviolet or infrared radiation. However, because each atom has its own distinct set of energy levels (its fingerprint!), each atom has its own distinct emission spectrum.

31.4.2 Absorption spectra

As you know, atoms do not only emit photons; they also absorb photons. If a photon hits an atom and the energy of the photon is the same as the gap between two electron energy levels in the atom, then the electron can absorb the photon and jump up to the higher energy level.

If the atom has no energy level differences that equal the incoming photon's energy, it cannot absorb the photon, and can only scatter it.

Using this effect, if we have a source of photons of various energies we can obtain the **absorption spectra** for different materials. To get an absorption spectrum, just shine white light on a sample of the material that you are interested in. White light is made up of all the different wavelengths of visible light put together. In the absorption spectrum, the energy levels corresponding to the absorbed photons show up as black lines because the photons of these wavelengths have been absorbed and don't show up. Because of this, the absorption spectrum is the exact *inverse* of the emission spectrum. Look at the two figures below. In figure 31.6 you can see the emission lines of hyrodrogen. Figure 31.7 shows the absorption spectrum. It is the exact opposite of the emission spectrum! Both emission and absorption techniques can be used to get the same information about the energy levels of an atom.

Figure 31.6: *Emission* spectrum of Hydrogen.

Figure 31.7: *Absorption* spectrum of Hydrogen.

Worked Example 191: Absorption

Question: I have an unknown gas in a glass container. I shine a bright white light through one side of the container and measure the spectrum of transmitted light. I notice that there is a black line (*absorption* line) in the middle of the visible red band at 642 nm. I have a hunch that the gas might be hydrogen. If I am correct, between which 2 energy levels does this transition occur? (Hint: look at figure 31.5 and the transitions which are in the visible part of the spectrum.)

Answer

Step 1 : What is given and what needs to be done?

We have an absorption line at 642 nm. This means that the substance in the glass container absorbed photons with a wavelength of 642 nm. We need to calculate which 2 energy levels of hydrogen this transition would correspond to. Therefore we need to know what energy the absorbed photons had.

Step 2 : Calculate the energy of the absorbed photons

$$
\begin{aligned}
E &= \frac{hc}{\lambda} \\
&= \frac{(6{,}63 \times 10^{-34}) \times (3 \times 10^{8})}{642 \times 10^{-9}} \\
&= 3{,}1 \times 10^{-19} \ \text{J}
\end{aligned}
$$

The absorbed photons had energy of $3{,}1 \times 10^{-19}$

Step 3 : Find the energy of the transitions resulting in radiation at visible wavelengths

Figure 31.5 shows various energy level transitions. The transitions related to visible wavelengths are marked as the transitions beginning or ending on Energy Level 2. Let's find the energy of those transitions and compare with the energy of the absorbed photons we've just calculated.

Energy of transition (absorption) from Energy Level 2 to Energy Level 3:

$$
\begin{aligned}
E_{2_3} &= E_2 - E_3 \\
&= 16{,}3 \times 10^{-19} \text{ J} - 19{,}4 \times 10^{-19} \text{ J} \\
&= -3{,}1 \times 10^{-19} \text{ J}
\end{aligned}
$$

Therefore the energy of the photon that an electron must absorb to jump from Energy Level 2 to Energy Level 3 is $3{,}1 \times 10^{-19}$ J. (NOTE: The minus sign means that *absorption* is occurring.)

This is the same energy as the photons which were absorbed by the gas in the container! Therefore, since the transitions of all elements are unique, we can say that the gas in the container is hydrogen. The transition is absorption of a photon between Energy Level 2 and Energy Level 3.

31.4.3 Colours and energies of electromagnetic radiation

We saw in the explanation of why the sky is blue that different *wavelengths* or *frequencies* of light correspond to different *colours* of light. The table below gives the wavelengths and colours for light in the visible spectrum:

Colour	Wavelength range (nm)
violet	390 - 455
blue	455 - 492
green	492 - 577
yellow	577 - 597
orange	597 - 622
red	622 - 780

Table 31.2: Colours and wavelengths of light in the visible spectrum.

We also know that the energy of a photon of light can be found from:

$$
E = hf = \frac{hc}{\lambda}
$$

Therefore if we know the frequency or wavelength of light, we can calculate the photon's energy and vice versa.

Activity :: Investigation : Frequency,wavelength and energy relation

Refer to table 31.2: Copy the table into your workbook and add two additional columns.

1. In the first new column write down the lower and upper frequencies for each colour of light.

2. In the second column write down the energy range (in Joules) for each colour of light.

Questions

1. Which colour of visible light has the highest energy photons?

2. Which colour of visible light has the lowest energy photons?

Worked Example 192: Colours of light

Question: A photon of wavelength 500 nm is emitted by a traffic light.

1. What is the energy of the photon?

2. What is the frequency of the photon?

3. Use table 31.2 to determine the colour of the light.

Answer

Step 1 : What information is given and what do we need to do find?

We are given $\lambda = 500 \times 10^{-9}$ m and we need to find the photon's *energy*, *frequency* and *colour*.

Step 2 : Use the equation $E = \frac{hc}{\lambda}$ to find the photon's energy

$$
\begin{aligned}
E &= \frac{hc}{\lambda} \\
&= \frac{(6{,}63 \times 10^{-34}) \times (3 \times 10^{8})}{500 \times 10^{9}} \\
&= 3{,}98 \times 10^{-19} \text{ J}
\end{aligned}
$$

The energy of the photon is $3{,}98 \times 10^{-19}$ J.

Step 3 : We know the energy of the photon, now we can use $E = hf$ to solve for the frequency

$$
\begin{aligned}
E &= hf \\
f &= \frac{E}{h} \\
&= \frac{3.98 \times 10^{-19}}{6{,}63 \times 10^{-34}} \\
&= 6 \times 10^{14} \text{ Hz}
\end{aligned}
$$

The frequency of the photon is 6×10^{14} Hz

Step 4 : Use the table to find the colour of light

The wavelength given in the question is 500 nm. We can see in the table that green light has wavelengths between 492 - 577 nm. Therefore 500 nm is in this range and the colour of the light is **green**.

Worked Example 193: Colours and energies of light

Question: I have some sources which emit light of the following wavelengths:

1. 400 nm,

2. 580 nm,

3. 650 nm,

4. 300 nm.

What are the colours of light emitted by the sources (see table 31.2) and which source emits photons with the highest energy and which with the lowest energy?

Answer

Step 1 : What information is given, and what do we need to do?

4 wavelengths of light are given and we need to find their *colours*.

We also need to find which colour light photons have the highest energy and which one has the lowest energy.

Step 2 : To find the colours of light, we can compare the wavelengths to those given in table 31.2

1. 400 nm falls into the range for *violet* light (390 - 455 nm).

2. 580 nm falls into the range for *yellow* light (577 - 597 nm).

3. 650 nm falls into the range for *red* light (622 - 780 nm).

4. 300 nm is not shown in the table. However, this wavelength is just a little shorter than the shortest wavelength in the violet range. Therefore 300 nm is *ultraviolet*.

Step 3 : To find the colour of the light whose photons have the highest and lowest energies respectively, we need to calculate the energies of all the photons

We know $E = \frac{hc}{\lambda}$

For 400 nm:

$$
\begin{aligned}
E &= \frac{hc}{\lambda} \\
&= \frac{(6{,}63 \times 10^{-34}) \times (3 \times 10^{8})}{400 \times 10^{-9}} \\
&= 4{,}97 \times 10^{-19} \text{ J}
\end{aligned}
$$

For 580 nm:

$$
\begin{aligned}
E &= \frac{hc}{\lambda} \\
&= \frac{(6{,}63 \times 10^{-34}) \times (3 \times 10^{8})}{580 \times 10^{-9}} \\
&= 3{,}43 \times 10^{-19} \text{ J}
\end{aligned}
$$

For 650 nm:

$$
\begin{aligned}
E &= \frac{hc}{\lambda} \\
&= \frac{(6{,}63 \times 10^{-34}) \times (3 \times 10^{8})}{650 \times 10^{-9}} \\
&= 3{,}06 \times 10^{-19} \text{ J}
\end{aligned}
$$

For 300 nm:

$$
\begin{aligned}
E &= \frac{hc}{\lambda} \\
&= \frac{(6{,}63 \times 10^{-34}) \times (3 \times 10^{8})}{300 \times 10^{-9}} \\
&= 6{,}63 \times 10^{-19} \text{ J}
\end{aligned}
$$

Therefore, the photons with the highest energy are the **ultraviolet** photons.

The photons with the lowest energy are from light which is **red**.

31.4.4 Applications of emission and absorption spectra

The study of spectra from stars and galaxies in astronomy is called *spectroscopy*. Spectroscopy is a tool widely used in astronomy to learn different things about astronomical objects.

Identifying elements in astronomical objects using their spectra

Measuring the spectrum of light from a star can tell astronomers what the star is made of! Since each element emits or absorbs light only at particular wavelengths, astronomers can identify what elements are in the stars from the lines in their spectra. From studying the

spectra of many stars we know that there are many different types of stars which contain different elements and in different amounts.

Determining velocities of galaxies using spectroscopy

You have already learned in Chapter 24 about the Doppler effect and how the frequency (and wavelength) of sound waves changes depending on whether the object emitting the sound is moving *towards* or *away* from you. The same thing happens to electromagnetic radiation. If the object emitting the light is moving *towards* us, then the wavelength of the light appears shorter (called **blue-shifted**). If the object is moving *away* from us, then the wavelength of its light appears stretched out (called **red-shifted**).

The Doppler effect affects the spectra of objects in space depending on their motion relative to us on the earth. For example, the light from a distant galaxy, which is moving away from us at some velocity, will appear red-shifted. This means that the emission and absorption lines in the galaxy's spectrum will be shifted to a longer wavelength (lower frequency). Knowing where each line in the spectrum would normally be if the galaxy was not moving, and comparing to their red-shifted positions, allows astronomers to precisely measure the velocity of the galaxy relative to the earth!

Global warming and greenhouse gases

The sun emits radiation (light) over a range of wavelengths which are mainly in the visible part of the spectrum. Radiation at these wavelengths passes through the gases of the atmosphere to warm the land and the oceans below. The warm earth then radiates this heat at longer infrared wavelengths. Carbon-dioxide (one of the main greenhouse gases) in the atmosphere has energy levels which correspond to the infrared wavelengths which allow it to absorb the infrared radiation. It then also emits at infrared wavelengths in all directions. This effect stops a large amount of the infrared radiation getting out of the atmosphere, which causes the atmosphere and the earth to heat up. More radiation is coming in than is getting back out.

the sun emits short wavelength radiation which penetrates the atmosphere

CO_2 molecules absorb and re-emit the infrared radiation in all directions heating the atmosphere

earth radiates long wavelength infrared radiation
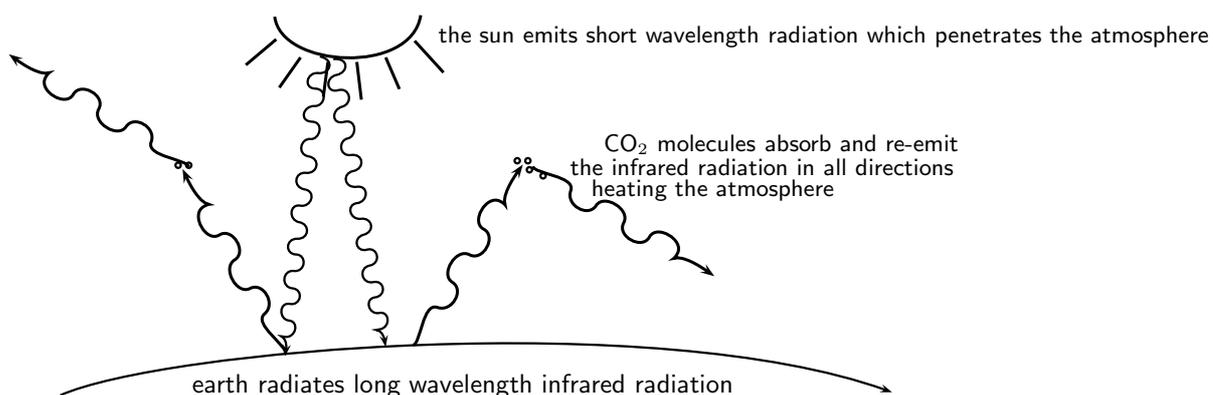
Therefore increasing the amount of greenhouse gases in the atmosphere increases the amount of trapped infrared radiation and therefore the overall temperature of the earth. The earth is a very sensitive and complicated system upon which life depends and changing the delicate balances of temperature and atmospheric gas content may have disastrous consequences if we are not careful.

Activity :: Investigation : The greenhouse effect
In pairs try to find the following information (e.g. in books, on the internet) and report back to the class in a 5 minute presentation which includes the following:

1. What other gases besides carbon dioxide are responsible for the greenhouse effect?

2. Where do greenhouse gases come from? (are they human-made or natural?)

3. Investigate one serious side-effect which could arise if the earth's temperature were to go up significantly. Present some ways in which this effect could be avoided.

?

Exercise: Emission and absorption spectra

1. Explain how atomic emission spectra arise and how they relate to each element on the periodic table.

2. How do the lines on the atomic spectrum relate to electron transitions between energy levels?

3. Explain the difference between of atomic absorption and emission spectra.

4. Describe how the absorption and emission spectra of the gases in the atmosphere give rise to the Greenhouse Effect.

5. Using table 31.2 calculate the frequency range for yellow light.

6. What colour is the light emitted by hydrogen when an electron makes the transition from energy level 5 down to energy level 2? (Use figure 31.5 to find the energy of the released photon.)

7. I have a glass tube filled with hydrogen gas. I shine white light onto the tube. The spectrum I then measure has an absorption line at a wavelength of 474 nm. Between which two energy levels did the transition occur? (Use figure 31.5 in solving the problem.)

31.5 Lasers

A laser is a device that produces a special type of light: all the laser photons are identical! They all have the *same* wavelength (and frequency), amplitude and phase. Since they all have the same wavelength, this means they all have the same *colour* and the light is called **monochromatic**. (*Note: mono* means "one" or "single" and *chromatic* means "colour".) This is very different to most other light sources which produce light with a range of wavelengths (e.g. white light from the sun consists of all the visible wavelengths.)

Laser light is highly directional and can be focused very well. This focus allows laser beams to be used over long distances, and to pack a lot of energy into the beam while still requiring reasonably small amounts of energy to be generated. Each centimetre of a typical laser beam contains many billions of photons. These special properties of laser light come from the way in which the laser photons are created and the energy levels of the material that makes up the laser. These properties make laser light extremely useful in many applications from CD players to eye surgery.
The term **LASER** stands for **L**ight **A**mplification by the **S**timulated **E**mission of **R**adiation.
This **stimulated emission** is different to the **spontaneous emission** already discussed earlier. Let's review the absorption and emission processes which can occur in atoms.
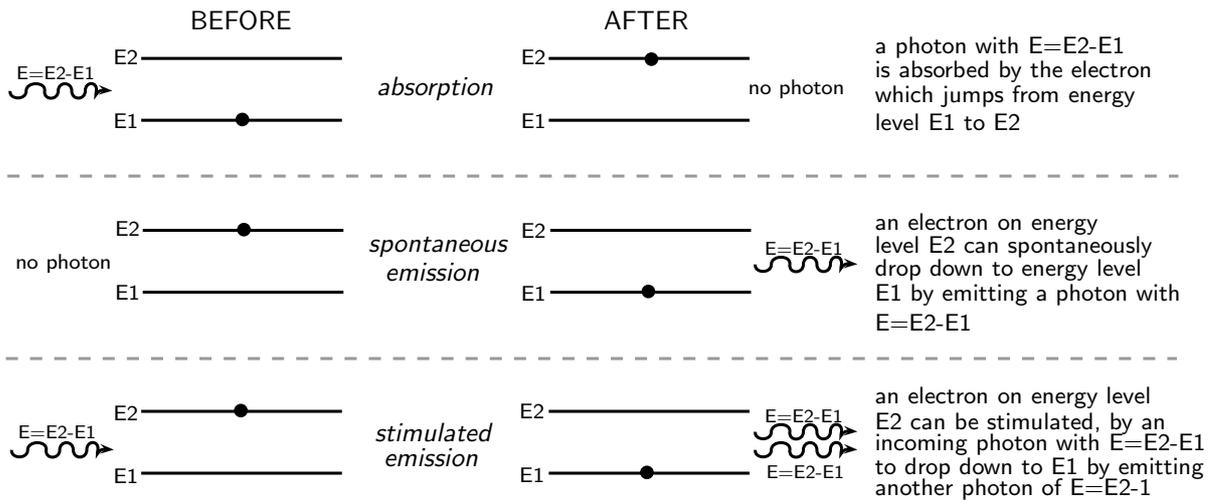
- **Absorption**: As you can see in the picture above, **absorption** happens when an electron jumps up to a higher energy level by *absorbing* a photon which has an energy equal to the energy difference between the two energy levels.

- **Spontaneous emission**: **Spontaneous emission** is when an electron in a higher energy level drops down to a lower energy level and a photon is emitted with an energy equal to the energy difference between the two levels. There is no interference in this process from outside factors. Usually spontaneous emission happens very quickly after an electron gets into an excited state. In other words, the lifetime of the excited state is very short. However, there are some excited states where an electron can remain in the higher energy level for a longer time than usual before dropping down to a lower level. These excited states are called **metastable states.**

- **Stimulated emission**: As the picture above shows, **stimulated emission** happens when a photon with an energy equal to the energy difference between two levels interacts with an electron in the higher level. This *stimulates* the electron to emit an identical photon and drop down to the lower energy level. This process results in two photons at the end.

Definition: Spontaneous Emission
Spontaneous emission occurs when an atom is in an unstable excited state and randomly decays to a less energetic state, emitting a photon to carry off the excess energy. The unstable state decays in a characteristic time, called the lifetime.

Definition: Meta-stable state
A meta-stable state is an excited atomic state that has an unusually long lifetime, compared to the lifetimes of other excited states of that atom. While most excited states have lifetimes measured in microseconds and nanoseconds (10^{-6} s and 10^{-9} s), meta-stable states can have lifetimes of milliseconds (10^{-3} s) or even seconds.

Definition: Stimulated emission
Stimulated emission occurs when a photon interacts with an excited atom, causing the atom to decay and emit another identical photon.

31.5.1 How a laser works

The important process involved in how a laser works is *stimulated emission* - as you can tell from what 'laser' stands for! You can imagine that stimulated emission can lead to more and more identical photons being released in the following way: Imagine we have an electron in an excited metastable state and it drops down to the ground state by emitting a photon. If this photon then travels through the material and meets another electron in the metastable excited state this will cause the electron to drop down to the lower energy level and another photon to be emitted. Now there are two photons of the same energy. If these photons then both move through the material and *each* interacts with another electron in a metastable state, this will result in them *each* causing an additional photon to be released. i.e. from 2 photons we then get 4, and so on! This is how laser light is produced.

Spontaneous Emission
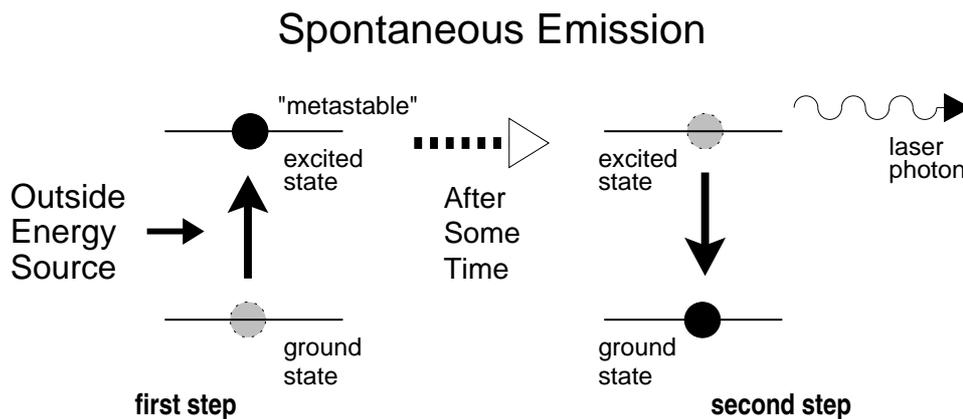
Figure 31.8: Spontaneous emission is a two step process, as shown here. First, energy from an external source is applied to an atom in the laser medium, raising its energy to an excited (metastable) state. After some time, it will decay back down to its ground state and emit the excess energy in the form of a photon. This is the first stage in the formation of a laser beam.

Stimulated Emission
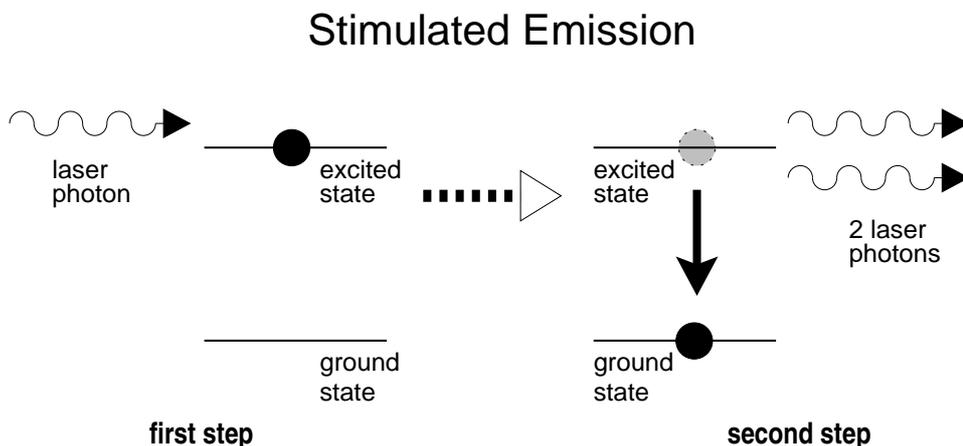
Figure 31.9: Stimulated emission is also a two step process, as shown here. First, a laser photon encounters an atom that has been raised to an excited state, just like in the case of spontaneous emission. The photon then causes the atom to decay to its ground state and emit another photon identical to the incoming photon. This is the second step in the creation of a laser beam. It happens many, many times as the laser photons pass through the optical cavity until the laser beam builds up to full strength.

This can only happen if there are many electrons in a metastable state. If most of the electrons are in the ground state, then they will just *absorb* the photons and no extra photons will be emitted. However, if more electrons are in the excited metastable state than in the ground state, then the process of stimulated emission will be able to continue. Usually in atoms, most of the electrons are in the lower energy levels and only a few are in excited states. When most

of the electrons are in the excited metastable state and only a few are in the ground state, this is called **population inversion** (the populations are *swapped* around) and this is when stimulated emission can occur. To start off the process, the electrons first have to be excited up into the metastable state. This is done using an external energy source.

Definition: Population inversion
Population inversion is when more atoms are in an excited state than in their ground state. It is a necessary condition to sustain a laser beam, so that there are enough excited atoms that can be stimulated to emit more photons.

---

usually most electrons are in the ground state

most electrons in excited metastable state = *population inversion*
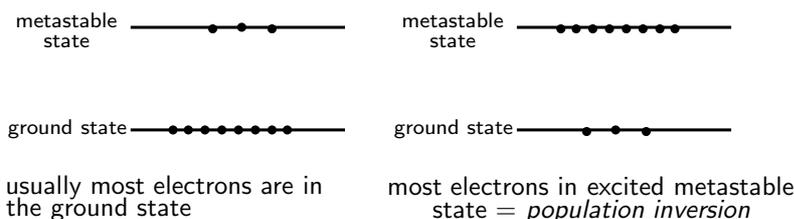
Therefore, materials used to make laser light *must* must have metastable states which can allow population inversion to occur when an external energy source is applied. Some substances which are used to make lasers are listed in table 31.3. You can see that gases (such as Helium-Neon mixture), liquids (such as dyes), and solids (such as the precious stone ruby) are all used to make lasers.

Material	Type	Wavelength	Uses
Helium–Neon	gas	632,8 nm	scientific research, holography
Argon ion	gas	488,0 nm	medicine,
Carbon dioxide	gas	10,6 μm	industry (cutting, welding), surgery
Helium–Cadmium	vapor	325 nm	printing, scientific research
Ruby	solid–state	694,3 nm	holography
Neodymium YAG (Yttrium Aluminium Garnet)	solid–state	1,064 μm	industry, surgery, research
Titanium–Sapphire	solid–state	650–1100 nm	research
Laser diode	semiconductor	375–1080 nm	telecommunications, industry, printing, CD players, laser pointers

Table 31.3: A selection of different lasers. The laser material and general type of each laser is given, along with typical wavelengths of the laser light they create. Examples of the real-world applications it is used for are also given. All these materials allow a population inversion to be set up.

Interesting Fact
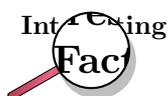
The first working laser, using synthetic ruby as the laser material, was made by Theodore H. Maiman at Hughes Research Laboratories in Malibu, California. Later in the same year the Iranian physicist Ali Javan, together with William Bennet and Donald Herriot, made the first gas laser using helium and neon. Javan received the Albert Einstein Award in 1993.

31.5.2 A simple laser

A laser consists of a number of different parts that work together to create the laser beam. Figure 31.10 shows the different parts of the laser, while Figure 31.11 shows how they create the laser beam.
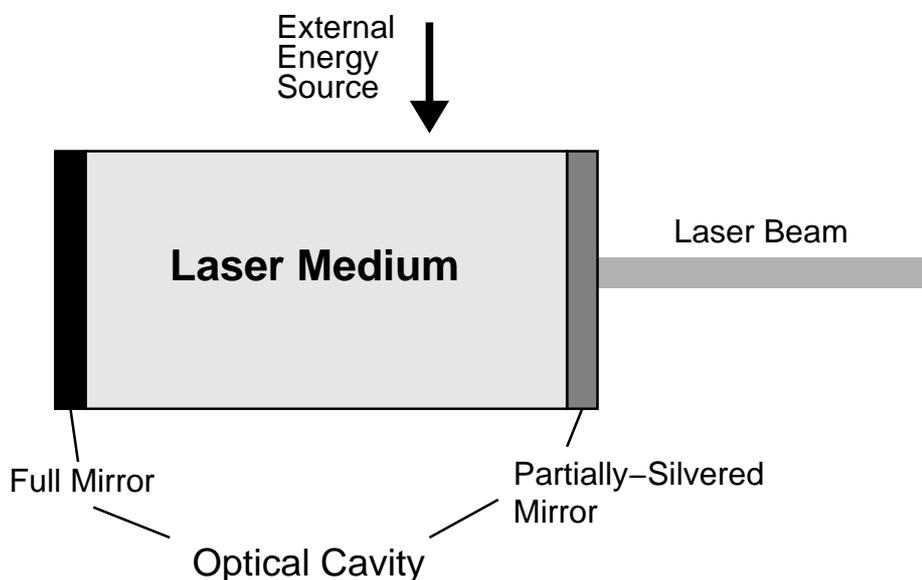
Figure 31.10: Diagram of a laser showing the main components.

The basis of the laser is the laser material. The laser material consists of the atoms that are used to create the laser beam. Many different materials can be used as laser material, and their energy levels determine the characteristics of the laser. Some examples of different lasers are shown in Table 31.3. The laser material is contained in the optical cavity.

Before the laser is turned on, all the atoms in the laser material are in their ground state. The first step in creating a laser beam is to add energy to the laser material to raise most of the electrons into an excited metastable state. This is called *pumping* the laser.

The creation of the laser beam starts through the process of *spontaneous emission*, shown in Figure 31.8. An electron drops down to the ground state and emits a photon with energy equal to the energy difference of the two energy levels. This laser photon is the beginning of the laser
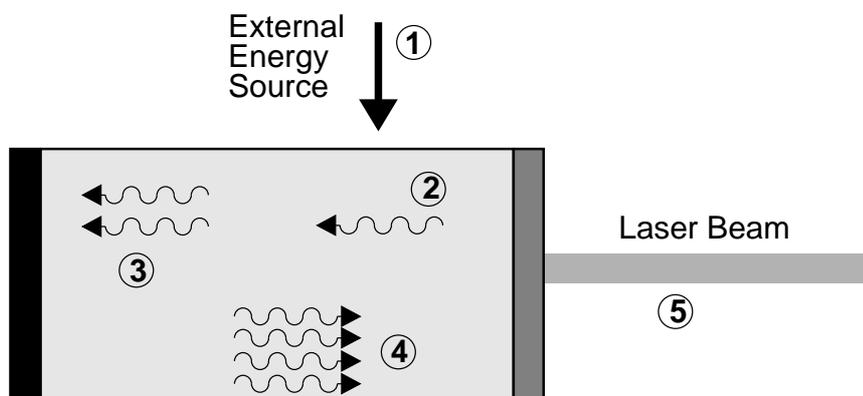
Figure 31.11: Diagram of a laser showing the process of creating a laser beam. (1) A source of external energy is applied to the laser medium, raising the atoms to an excited state. (2) An excited atom decays though spontaneous emission, emitting a photon. (3) The photon encounters another excited atom and causes it to decay through stimulated emission, creating another photon. (4) The photons bounce back and forth through the laser medium between the mirrors, building up more and more photons. (5) A small percentage of the photons pass through the partially-silvered mirror to become the laser beam we see.

beam.

Sometimes a laser photon runs into another excited electron. Then stimulated emission occurs and the electron drops down to the ground state and emits an additional identical photon as shown in Figure 31.9. Since the laser material typically has a large number of atoms, one laser photon passing through this material will rapidly cause a large number of photons just like it to be emitted.

The optical cavity keeps the laser photons inside the laser cavity so they can build up the laser beam. At each end is a concave mirror; one is a full mirror and one is a partial mirror. The full mirror is totally reflective. The partial mirror transmits a small amount of the light that hits it(less than 1%). The mirrors are carefully aligned so that photons that reflect off one mirror become "trapped", and bounce back and forth between the mirrors many times causing more and more stimulated emission. The photons that eventually escape through the partially-silvered mirror become the laser beam that we see.

As the photons bounce between mirrors, they continually pass through the laser material, stimulating those atoms to emit more photons. This creates an ever increasing beam of photons, all with the same characteristics, all traveling in the same direction. In this way, the optical cavity helps to amplify the original laser photons into a concentrated, intense beam of photons.

The laser cavity also helps to narrow the frequency range of laser light emitted. The distance between the two mirrors defines the cavity *mode* which only allows light of a narrow range of frequencies to continue being reflected back and forth. Light of other frequencies damped out. (This is just like in the chapter on the physics of music where a pipe of a certain length corresponds to a particular wavelength of sound.) Therefore only a narrow frequency of light can be emitted.

Interesting Fact
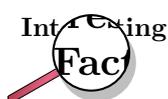
In 1953, Charles H. Townes and graduate students James P. Gordon and Herbert J. Zeiger produced the first maser, a device operating on similar principles to the laser, but producing microwave rather than optical radiation. Townes's maser was incapable of making a continuous beam. Nikolay Basov and Aleksandr Prokhorov of the former Soviet Union worked independently and developed a method of making a continuous beam using more than two energy levels. Townes, Basov and Prokhorov shared the Nobel Prize in Physics in 1964.

31.5.3   Laser applications and safety

Although the first working laser was only produced in 1958, lasers are now found in many household items. For example, lasers are well-known through their use as cheap laser pointers. However, lasers can be very dangerous to the human eye since a large amount of energy is focused into a very narrow beam. **NEVER POINT A LASER POINTER INTO SOMEBODY'S EYES** - **it can blind them.**

Other uses include:

- Semiconductor lasers which are small, efficient and cheap to make are found in CD players.

- He-Ne Lasers are used in most grocery shops to read in the price of items using their barcodes. This makes the cashiers' job much quicker and easier.

- High energy lasers are used in medicine as a cutting and welding tool. Eye surgery in particular make use of the precision of lasers to reattach the retinas of patients' eyes. The heat from cutting lasers also helps to stop the bleeding on a wound by burning the edges (called cauterising).

Activity :: Case Study : Uses of lasers

Do research in a library or on the Internet on one application of laser technology. Explain how the technology works by using a laser.

You will need to present your findings to the class in the form of a poster. You can think of any useful application, but to give you some ideas of where to start, some applications are listed below:

- laser printers
- laser communication and fibre optics
- optical storage
- using lasers as precision measurement tools
- your own ideas...

?

Exercise: Lasers

1. Explain what is meant by *spontaneous emission of radiation*.
2. Explain what is meant by *stimulated emission of radiation*.
3. List the similarities and differences between spontaneous emission of radiation and stimulated emission of radiation.
4. How is the light emitted by a laser different from the light emitted by a light bulb?
5. Describe using a simple diagram, how a laser works. Your description should include the following concepts: metastable state and population inversion.
6. Give examples of some materials that have been used for lasers. What do all these materials have in common?
7. Describe how the laser cavity affects:
   - increasing amplification
   - concentrating beam intensity
   - narrowing the frequency of the beam
8. List some applications of lasers.

31.6   Summary

1. Light of the correct frequency can emit electrons from a metal. This is called the photoelectric effect.

2. A metal has a work function which is the minimum energy needed to emit an electron from the metal.

3. Emission spectra are formed by glowing gases. The pattern of the spectra is characteristic of the specific gas.

4. Absorption spectra are formed when certain frequencies of light is absorbed by a material.

5. Lasers are devices that produce a special type of light that has many uses.

6. Lasers have many uses, including being used in CD and DVD players, to cut material, in surgery, in printing, in telecommunications and as laser pointers.

31.7   End of chapter exercise

1. What is the photoelectric effect?

2. Calculate the energy of a photon of red light with a wavelength of 400 nm.

3. Will ultraviolet light with a wavelenth of 990 nm of be able to emit electrons from a sheet of calcium with a work function of 2,9 eV?

4. What does the acronym LASER stand for?

5. Name three types of lasers and their uses.

6. Write a short essay on the benefits lasers have had on modern society.

Appendix A

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